

**PART ONE:** Answer all 30 questions by circling the correct answer:  
(75%)

1.  $R = 48x - 2x^2$  where  $R$  is the total revenue in dinars, and  $x$  is the output. At what output is total revenue a maximum?

- a) 12  
c) 2

- b) 48  
d) 4

$$\frac{dR}{dx} = 48 - 4x$$

$$48 - 4x = 0$$

2. What price will be charged for the output which maximizes total revenue in question (1):

- a) 24 dinars  
c) 12 dinars

- b) 4 dinars  
d) 48 dinars

3. If a demand curve is  $p = 40 - 0.1q$ , and demand increases by 30 at each price level, then the equation of the new demand curve is:

- a)  $p = 70 - 0.1q$   
c)  $p + 0.1q = 43$

- b)  $p - 0.1q = 43$   
d)  $p = 37 - 0.1q$

$$p = 40 - 0.1q + 3$$

$$40 - 0.1q + 3 = 43$$

$$40 - 0.1q = 40 - 3$$

$$-0.1q = -3$$

$$q = 30$$

4. A firm plans to sell at prices which are 45% markup on retail. If the firm sells  $x$  dinars and the firm gives commission of 20 piasters for every dinar of sales, then the variable cost is:

- a)  $0.75x$   
c)  $0.55x$

- b)  $0.2x$   
d)  $0.65x$

3, 55x + 20

PART FIVE: The break-even point of the cost-sales equation:  $y = 0.6x + 120$  is:

- a) 120  
c) 200

- b) 300  
d) 450

$$\frac{120}{1 - 0.6}$$

$$\frac{120}{0.4}$$

6. The equation  $(2x + 3)(3x - 1) = 6$  has a:

- a) no solutions  
c) two solutions

- b) one solution  
d) three solutions

$$6x^2 - 2x + 3x - 3 = 6$$
  
~~2x~~  
$$6x^2 + x - 9 = 0 \quad | \quad a = 6, b = 1, c = -9$$
  
$$1 - 4(6)(-9)$$
  
$$1 + 42 = 83$$

7. If the total cost of producing  $x$  units is  $C(x) = 3^{0.2x} + \log(5x + 100) + 8$  dinars, then the fixed costs are:

- a) 8 dinars  
c)  $\log 5$  dinars

- b) 11 dinars  
d)  $3^{0.2}$  dinars

$$C(x) = 3^{0.2x} + \log(5x + 100) + 8$$
  
$$1 + \log(5x + 100) = 8$$
  
$$1 + 2 + 8 = 11$$

8. The vertex of the parabola  $y = x^2 + 4x + 8$  is:

- a)  $(-2, 4)$   
c)  $(-2, 8)$

- b)  $(2, 20)$   
d)  $(2, 8)$

$$a = 1, b = 4, c = 8$$
  
$$-\frac{4}{2} = (-2, 8)$$

~~4~~  
$$4 + (-8) + 8$$

9. If the relationship between total cost  $y$  piasters and number of units made  $x$  is linear, and if cost increases by 20 piasters for each additional unit made, and total cost of 10 units is 30 dinars, the equation of cost-output relationship is:

- a)  $y = 20x - 170$   
c)  $y = 0.2x + 28$

- b)  $y = 20x + 2800$   
d)  $y = 0.2x + 2998$

$$20x + 30$$

10. The inverse of  $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$  is  $\begin{pmatrix} 8 & -3 \\ -1 & 2 \end{pmatrix}$ . The cost in dollars of producing  $x$  units is  $\begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}$

a)  $\begin{pmatrix} 0.8 & -0.6 \\ -0.2 & 0.4 \end{pmatrix}$

b)  $\begin{pmatrix} 0.8 & -0.6 \\ 0.2 & -0.4 \end{pmatrix}$

c)  $\begin{pmatrix} 0.8 & 0.6 \\ 0.2 & 0.4 \end{pmatrix}$

d)  $\begin{pmatrix} -0.8 & 0.6 \\ -0.2 & 0.4 \end{pmatrix}$

11. The equation of the line that passes through  $(5,0)$  and  $(5,1)$  is:

a)  $y - 6x = 5$

b)  $y - 5 = 0$

c)  $x - 5 = 0$

d)  $x + 5 = 0$

$$\begin{aligned} y - 0 &= 1(x - 5) \\ y &= x - 5 \end{aligned}$$

12. The system of equations:  $y - x - 1 = 0$  and  $4x - 2y = 2$  has:

a) no solutions

b) solution  $= (0,1)$

c) solution  $= (2,3)$

d) solution  $= (0.5,1.5)$

$$\begin{aligned} y - x - 1 &= 0 \\ 4x - 2y &= 2 \end{aligned}$$

$$\begin{aligned} y - x - 1 &= 0 \\ -2y &= -2 \end{aligned}$$

$$\begin{aligned} y &= x + 1 \\ 4x - 2(x + 1) &= 2 \\ 2x - 2 &= 2 \\ 2x &= 4 \\ x &= 2 \end{aligned}$$

13. If the demand curve is  $p = 40 - 0.1q$  and the supply curve is  $5p - q = 50$ , the equilibrium point is:

a)  $(30,100)$

b)  $(100,30)$

c)  $(30,30)$

d)  $(100,100)$

$$\begin{aligned} 5p - q &= 50 \\ 5(40 - 0.1q) - q &= 50 \\ 200 - 5q - q &= 50 \\ 200 - 6q &= 50 \\ -6q &= -150 \\ q &= 25 \end{aligned}$$

14. Suppose the economy of a country has an *Oil* industry and an *Agricultural* industry, with technological matrix:

$$T = \begin{pmatrix} O & A \\ 0.5 & 0.5 \\ 0.2 & 0.6 \end{pmatrix} \begin{matrix} Oil \\ Agricultural \end{matrix}$$

If a surplus (external demand) of 300 units of *Oil* products and 200 units of *Agricultural* products are desired, then the gross production (output) of the *Agricultural* industry is:

- a) 2300  
c) 1900

- b) 2200  
d) 1600

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0.5 & 0.5 \\ 0.2 & 0.6 \end{pmatrix} = \begin{pmatrix} 0.5 & -0.5 \\ -0.2 & 0.4 \end{pmatrix} \begin{pmatrix} 300 \\ 200 \end{pmatrix}$$

15. How much does the *Oil* industry take from the *Agricultural* industry (from question 14)?

- a) 320  
c) 800

- b) 440  
d) 1100

$$\begin{pmatrix} 0.5 & 0.5 \\ 0.2 & 0.6 \end{pmatrix} \begin{pmatrix} 300 \\ 200 \end{pmatrix} = \begin{pmatrix} 220 \\ 200 \end{pmatrix}$$

$$\begin{pmatrix} 0.5 & 0.5 \\ 0.2 & 0.6 \end{pmatrix} \begin{pmatrix} 300 \\ 200 \end{pmatrix} = \begin{pmatrix} 160 \\ 140 \end{pmatrix}$$

16. If  $\log(4x) - \log(x+1) = 1$ , then  $x =$

- a)  $-3/5$   
c)  $-2$

- b)  $1/9$   
d)  $-5/3$

$$\log \frac{4x}{x+1} = 1$$

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$$\log \frac{4x}{x+1} = 1$$

$$(4x)(x+1) = 10^1$$

17. If  $g(x) = (3x^2+1)^4$ , then  $g'(x) =$

- a)  $24x(3x^2+1)^3$   
c)  $4(3x^2+1)^3$

- b)  $4(6x)^3$   
d)  $12x(3x^2+1)^3$

$$4(3x^2+1)^3 (6x)$$

$$x^2 \left( \frac{4x}{x+1} \right) - (10)$$

$$4x - 10x^2 - 10$$

~~18.~~ If  $y = \log e^x$ , then  $y' =$

- a) 0      b)  $1/\ln 10$   
c)  $1/\log e$       d)  $x/\ln 10$

~~19.~~ The slope of the tangent line to the curve  $f(x) = 2x^3 - 6x + 1$  at  $x = -1$  is:

- a) 0      b) 5  
c) -12      d) -3

$$\begin{aligned}f'(x) &= 6x^2 - 6 \\f'(-1) &= 6(-1)^2 - 6 \\6 - 6 &= 0\end{aligned}$$

~~20.~~  $g(x) = 8 + 2x^3 - 6x$  has a minimum point at:

- a)  $x = -3$       b)  $x = 3$   
c)  $x = 1$       d)  $x = -1$

$$\begin{aligned}f'(x) &= 6x^2 - 6 \\6x^2 - 6 &= 0 \\6x^2 &= 6 \\x^2 &= 1 \\x &= \pm 1\end{aligned}$$

~~21.~~ If  $f = e^{2x+y}$ , then  $f_y =$

- a)  $e^2$       b)  $e^{2x+y}$   
c)  $2e^{2x+y}$       d)  $e^{2+y}$

$$f_y = e^{2x+y} \cdot 1$$

~~22.~~ The equation of the tangent line to the curve  $f(x) = x + e^{2x} + 1$  at  $x = 0$  is:

- a)  $y = 2x$       b)  $y = 2$   
c)  $y = 3x + 2$       d)  $y = 3x$

$$\begin{aligned}f(x) &= x + e^{2x} + 1 \\f(0) &= 1 + e^0 \cdot 2 \\1 + 2 &= 3\end{aligned}$$

$$f(x) = x + e^{2x} + 1$$

$$1 + 2 \cdot 2/2 \cdot 2 = 3.4$$

~~23.~~ If  $f(x,y) = xe^y + ye^x$ , then  $f_{xy} =$

- a)  $e^y + e^x$   
c)  $e^{x+y}$

- b)  $e^y + ye^x$   
d)  $ye^x$

$$f(x) = e^y \cdot 1 + x \cdot e^y f(e^x \cdot 0)$$

$$\cancel{e^y} + x + \cancel{y} \in$$

$$f_{xy} = e^y + e^x \cdot 1 + \cancel{ye^x}$$

$$e^y + e^x + \cancel{y}$$

~~24.~~ If  $g(x,y) = x/y$ , then  $g_y =$

- a)  $1/y^2$   
c)  $x/y^2$

- b)  $-1/y$   
d)  $-x/y^2$

$$\frac{x}{y}$$

$$g_y = \frac{(y \cdot 0) - x \cdot 1}{y^2}$$

$$-\frac{x}{y}$$

~~25.~~ If  $2x^2 + 2y - 4 = 0$ , then  $y' =$

- a)  $2x$   
c)  $4x + 2$

- b)  $-2x$   
d)  $4x$

$$2x^2 + 2y - 4 = 0$$

$$2x^2 + 2y = 4$$

$$2x^2 = 4 - 2y$$

$$x^2 = 2 - y$$

$$x = \sqrt{2-y}$$

$$2x^2 + 2y = 0$$

$$2x^2 = -2y$$

$$x^2 = -y$$

$$x = \sqrt{-y}$$

~~26.~~ The cost-sales equation is  $y = 0.8x + 45$ . If variable costs other than the cost of goods were 12% of sales, then the mark-up is:

- a) 33%  
c) 68%

- b) 32%  
d) 43%

27. If 1 dinar is deposited at 12% interest compounded quarterly, what will be the amount after 2 years and 3 months?

a)  $(1.12)^{2.25}$  dinars  
 c)  $(1.03)^9$  dinars

b)  $(1.04)^9$  dinars  
 d)  $(1.01)^{27}$  dinars

$$P \left(1 - \frac{1}{m}\right)^{2 \times 4} \\ 1 \left(1 - \frac{0.12}{4}\right)^{2 \times 4}$$

$$1 \left(1 - \frac{0.12}{4}\right)^{2 \times 4}$$

28. The function  $f(x,y) = 2x^2 + 4x + 6y^2 - 24y - 20$  has a:

- a) maximum when  $x = -1$  and  $y = 2$   
 b) minimum when  $x = -1$  and  $y = 2$   
 c) minimum when  $x = 0$  and  $y = 0$   
 d) no maximum and no minimum

$$f_x = 4x + 4$$

$$f_y = 12y - 24$$

$$f_{xx} = 4$$

$$f_{yy} = 12$$

$$f_{xy} = 0$$

If  $5^{2x} = 15$ , then  $x =$

- a)  $\frac{1}{2} \ln 5$   
 c)  $\frac{1}{2}(1 + \log_5 3)$

$$4x + 4 = 0$$

$$4x = -4 \quad (4)(12) < 0$$

$$x = -1$$

$$12y - 24 = 0$$

$$12y = 24$$

$$y = 2$$

$$(-1, 2)$$

29.

- b)  $\frac{1}{2} \ln 3$   
 d)  $\frac{1}{2}(1 + \log_3 5)$

~~$\log 5$~~

~~$\log 5$~~

$$\log 5 - \log 15$$

$$2x \log 5 - \log 3 + \log 5$$

$$2x \log 5 - 1.196$$

$$2x(\log 5) =$$

$$\ln \frac{5}{3} = \ln 1.6$$

$$2x \ln 5 = \ln 1.6$$

$$2x \ln 5 = 2.408$$

$$\frac{3.21}{3.21} x = \frac{2.408}{3.21}$$

30.

The steady state vector for the transition matrix  $\begin{pmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{pmatrix}$  is:

- a)  $(0.3 \ 0.7)$   
 c)  $(0.4 \ 0.6)$

- b)  $(0.6 \ 0.4)$   
 d)  $(0.8 \ 0.2)$

$$0, 8 u$$

$$0, 8 u = 5 u$$

$$0.7u + 0.2v = u$$

$$0.7u + 0.2v = u$$

$$0.7u + 0.2v = u$$

$$0.7u - 0.2v = u$$

$$-0.2u = u - 0.7u$$

$$0.7u + 0.2v = u$$

**PART TWO:** Answer all TWO questions... Explain every step... Show all your work on this paper.

I (13%). The cost of making a car is two thousands dinars per car, and the fixed costs are three thousands dinars. The demand for the product at a price  $p$  thousands dinars per car is given by:  $x = 20 - 5p$

a) Find the cost, revenue, and profit functions.

$$p = \frac{20-x}{5}$$

$$C(x) = 2000x + 3000$$

$$-\frac{1}{5}x - 20 - 5p$$

$$R(x) = x(p)$$

$$-\frac{1}{5}x - 4 + p$$

$$\therefore x = 20 - 5p$$

$$\cancel{\frac{1}{5}x}$$

$$x - 20 = -5p \implies p = -\frac{1}{5}x + 4 \quad -\frac{1}{5}x + 4 = p$$

$$R(x)$$

$$P = R(x) - C(x) \implies -\frac{x^2}{5} + 4x$$

b) Find the maximum profit?

$$a = -\frac{1}{5}, b = \cancel{-4} \quad -\frac{x^2}{5} + 4x - 2000x + 3000$$

(-1)

$$\frac{+1996}{-\frac{1}{5}} = \underline{1996} = -9980$$

$$+\frac{(-9980)}{5} = 1996(-9980) + 30000$$

$$-9984$$

$$(-9980 \times 19915096)$$

R

c) What unit price should be charged to get the maximum profit?

$$(1996) - 4(-\frac{1}{5})(-3000)$$

$$= -1996$$

ex 2

II (12%). A bakery makes bread in two sizes: small and medium. The cost in dinars of producing  $x$  pieces of the small size and  $y$  pieces of the medium size is given by:  $C(x,y) = 12x + 28y$ .  
 The demands for the two sizes are:  $p_1 = 6y - 6x$   
 $p_2 = 6x - 10y + 200$

where  $p_1$  is the price in dinars per piece for the small size, and  $p_2$  is the price in dinars per piece for the medium size.

a) Determine how many pieces of each size the bakery should make to maximize profit.

$$C(x) = (6y - 6x) + 6x - 10y + 200 - 12x + 28y$$

$$2\cancel{y} - 4y + 200 - 12x$$

$$6y - 6x + 6x - 10y + 200 - (12x + 28y)$$

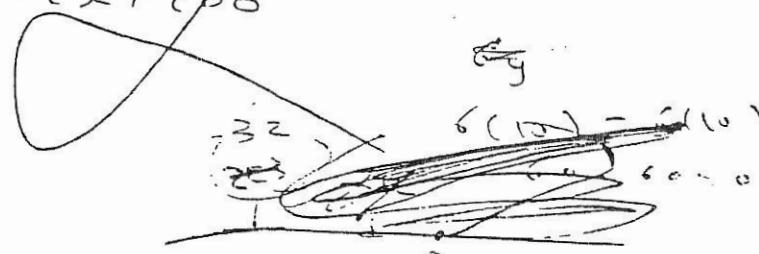
$$-4y + 200 - (12x + 28y)$$

$$-32y - 12x + 200$$

$$f_x = \cancel{-12} - 12$$

$$\cancel{f_y}:$$

$$f_y = -32$$



b) What is the maximum profit?

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